

$$f(x) = \arctan(x - \sqrt{1+x^2})$$

$$\begin{aligned} f' &= \frac{1}{1 + (x - \sqrt{1+x^2})^2} \cdot \left[1 - \frac{1}{2}(1+x^2)^{-\frac{1}{2}} \cdot 2x\right] \\ &= \frac{1}{1+x^2 - 2x\sqrt{1+x^2} + 1+x^2} \cdot \left(1 - \frac{x}{\sqrt{1+x^2}}\right) \\ &= \frac{1}{2x^2 - 2x\sqrt{1+x^2} + 2} \cdot \frac{\sqrt{1+x^2} - x}{\sqrt{1+x^2}} \\ &= \frac{1}{2(x^2 - x\sqrt{1+x^2} + 1)} \cdot \frac{\sqrt{1+x^2} - x}{\sqrt{1+x^2}} \cdot \frac{\sqrt{1+x^2}}{\sqrt{1+x^2}} \\ &= \frac{1}{2(x^2 - x\sqrt{1+x^2} + 1)} \cdot \frac{x^2 + 1 - x\sqrt{1+x^2}}{1+x^2} \\ &= \frac{x^2 - x\sqrt{1+x^2} + 1}{2(x^2 - x\sqrt{1+x^2} + 1)(1+x^2)} \\ &= \frac{1}{2+2x^2} \end{aligned}$$